ARTICLE NO. 80-4047

Instrument Modeling for Aerodynamic Coefficient Identification from Flight Test Data

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A large number of estimation methods can be used to compute aerodynamic coefficients of ballistically flown spinning vehicles. These estimation schemes often ignore possible phase shifts between measured data and the math model of the motion. Some of these problems occur because onboard instrumentation such as rate gyros or accelerometers behave like spring mass systems. If the vehicle motion is near the resonance frequency of the instrument, the output of the instrument is shifted in phase from the actual motion and probably the model representation of the motion. In order to demonstrate this problem, the aerodynamic coefficients of a spinning vehicle are computed from actual flight test data using a standard least-squares estimation scheme. Comparisons of observed and computed measurements of the vehicle motion are presented and the problem just described is shown to exist for these data. A modeling scheme is derived to compensate for the instrumentation induced changes in the data. The results obtained using the instrumentation modeling scheme are shown to be much better than those obtained using a standard scheme.

Nomenclature

A_y = acceleration in body-fixed y direction
A_z = acceleration in body-fixed z direction
$C_1,...,C_4$ = constant vectors associated with solution of linear vector differential equations
C_{y_0} = aerodynamic coefficient associated with force asymmetry in y direction
C_{z_0} = aerodynamic coefficient associated with force asymmetry in z direction
C_{y_β} = partial derivative of force coefficient in y direction with respect to β
C_{z_α} = partial derivative of force coefficient in z direction with respect to α
C_{m_0} = aerodynamic coefficient associated with asymmetrical moment in m direction
C_{n_0} = aerodynamic coefficient associated with asymmetrical moment in n direction
C_{m_α} = partial derivative of moment coefficient in m direction with respect to α
C_{n_β} = partial derivative of moment coefficient in n direction with respect to β
C_{m_q} = partial derivative of moment coefficient in m direction with respect to q
C_{n_r} = partial derivative of moment coefficient in n direction with respect to r
d = vehicle base diameter
F_y = lateral force in y direction
F_z = lateral force in z direction
G = vector function of observation-state relationships
I = lateral moment of inertia
I_x = roll moment of inertia
m = mass of the vehicle
M_y = vehicle pitching moment
M_z = vehicle yawing moment
n = number of observations
p = vehicle spin rate
q = vehicle pitching rate
q' = dynamic pressure
r = vehicle yawing rate
S = vehicle reference area
t = independent variable, time
u = body-fixed velocity component along axial direction
v = body-fixed lateral velocity component along y direction
V = total velocity of vehicle
w = body-fixed lateral velocity component along z direction
W = weighting matrix in least-squares cost function
y = vector of observations
z = auxiliary state variable associated with differential equation modeling the instrument
α = angle of attack
β = angle of sideslip
ϵ = error between observed and computed measurements
λ = damping term associated with second-order linear differential equation
Λ = matrix of damping terms for instrument modeling equation
ω = frequency terms associated with second-order linear differential equation
ω_1,ω_2 = vehicle lateral motion frequencies for a spinning vehicle
Ω = matrix of frequency terms for instrument modeling equation

Subscripts

i = evaluated at t_i
0 = evaluated at the initial time

Received March 5, 1979; revision received July 9, 1979.

Index categories: Entry Vehicle Dynamics and Control; LV/M Dynamics and Control.

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RG = rate gyro
LA = lateral accelerometer

Superscripts

T = transpose
$(\dot{\ })$ = derivative, $d/dt(\ )$

Introduction

AFTER the flight test of any vehicle the measurement data obtained from the flight are used to reconstruct the vehicle trajectory and to compute the aerodynamic coefficients for the vehicle. The accuracy with which the aerodynamic coefficients can be computed from a flight test substantially affects the ability to refine prediction schemes and understand the various aerodynamic effects on a flight test vehicle.

In general the computation of aerodynamic coefficients is accomplished by defining a state model and a state observation relationship. Both models may contain the aerodynamic coefficients. These coefficients, along with the initial conditions of the state variables, are then computed such that the solution to the model matches the actual data in some statistical sense. This approach is well documented and is discussed, for example, in Refs. 1, 2, and 3. There is, however, an additional part of the problem that has not appeared in the literature. Onboard instruments such as rate gyros and accelerometers essentially behave like spring mass systems (second-order differential equations) and as a result can introduce phase shifts and amplitude changes in the data.[4] These changes depend upon the natural frequency and damping of the instrument as well as the motion frequency of the flight test vehicle. For some flight tests the frequency of the motion of the vehicle may be near the natural frequency of the instrument. This is especially true when vehicles which will not be reecovered are flight tested. The instrumentation is often selected because of cost considerations rather than the desired quality of the data. Thus inexpensive "off the shelf" instruments with low natural frequencies may be used instead of high quality but expensive instrumentation packages. It is then necessary for the flight test data analyst to compensate for the large shifts in the flight test data relative to the actual motion of the vehicle.

Not only are these single instrument phase shifts important, but the relative effects between different instruments are very important. Since the natural frequencies of different instruments may not be identical, the phase shift for rate gyros will probably be different from that for lateral accelerometers. If the data from both instruments are used simultaneously to compute the coefficients, the phase shifts due to the different natural frequencies are different and must be considered in the coefficient identification scheme. Another problem occurs if more than one frequency appears in the motion. This is usually the case when the lateral modes of motion are coupled, as is discussed in Ref. 5. If only one frequency appeared in the data, it would be possible to ignore amplitude modulations and shift one set of data in time so that its frequencies would be aligned with data from other instruments. When two frequencies are present, however, they will be shifted relative to each other by each instrument. The data cannot be shifted in time to account for the relative shift of the two frequencies in one data source.

The proposed solution to this problem models the output of each set of instruments (gyros, accelerometers, etc.) as the solution to a second-order, linear, constant-coefficient differential equation. This allows the data from different instruments to be shifted relative to each other by the differing natural frequencies or constant coefficients associated with each differential equation. It also allows several frequencies associated with the state model to be shifted relative to each other as is the case with the actual instrument. Thus, if the natural frequencies and damping ratios of the instruments are known, they are specified in the aerodynamic coefficient identification scheme. The values of the state variables as provided by the vehicle equations of motion are used as the forcing term for the second-order differential equation model for the instrument. The result of the solution to this equation is compared with the actual data and the aerodynamic coefficients and the initial conditions to the equations of motion are selected to make this solution match the actual data in a least-squares sense. This incorporates the instrument model directly in the overall system model. If the natural frequency and damping ratio of the instrument are not known, they may easily be estimated in the same manner that the aerodynamic coefficients are estimated. This models the phase shifts directly and also accounts for multiple frequencies in the motion of the vehicle.

In order to demonstrate the effectiveness of the instrument modeling scheme, it is used to compute the dynamical motion and the aerodynamic coefficients of a spinning vehicle from actual flight test data. The state model consists of the four coupled first-order differential equations shown in Ref. 6 which govern the lateral motion of a spinning vehicle. A least-squares technique described in Ref. 7 is used to estimate the aerodynamic coefficients. For this example, the motion frequency is approximately two-thirds of the natural frequency of the instrumentation. The results show that when the instrument modeling scheme is used, good agreement between the data and model is obtained. This is not the case when the instrument modeling scheme is not used. Thus, when the motion frequency is fairly large relative to the instrumentation frequency, the proposed scheme to compensate for this phase shift improves the relative fit between the observed and computed measurements of the vehicle motion.

Estimation Problem

The problem of reconstructing the trajectory and estimating the aerodynamic coefficients of a spinning vehicle from flight test data has been discussed extensively in the literature. References 2 and 3 describe numerous estimation techniques applied to this problem. For most approaches, a dynamical model is developed, a state observation equation is derived, and the initial conditions of the equations of motion and the aerodynamic coefficients are computed so that the solution to the model matches the measured data in some statistical sense.

For the problem considered here, the lateral motion of a spinning vehicle flying a ballistic trajectory at a small angle of attack can be modeled by the following system of equations shown in Ref. 6.

$$
\begin{aligned}
\dot{v} &= -ur + wp + F_y/m \\
\dot{w} &= uq - vp + F_z/m \\
\dot{q} &= [(I - I_x)/I]pr + M_y/I \\
\dot{r} &= [(I_x - I)/I]pq + M_z/I
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
F_y &= q'S(C_{y_0} + C_{y_\beta}\beta) \\
F_z &= q'S(C_{z_0} + C_{z_\alpha}\alpha) \\
M_y &= q'Sd(C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}q) \\
M_z &= q'Sd(C_{n_0} + C_{n_\beta}\beta + C_{n_r}r)
\end{aligned}
\tag{2}
$$

and

$$
\alpha = w/V \qquad \beta = v/V \tag{3}
$$

If the dynamic pressure, velocity, and spin rate histories are known, these equations can be integrated to predict the lateral motion of the vehicle.

These equations are written with only linear terms in the force and moment expressions. The estimation scheme can, however, easily determine additional nonlinear terms if they are required to fit the data.

Next, the state observation equations must be obtained. If both rate gyro and lateral accelerometer data are available from the flight test, this equation may be expressed as

$$y_i = G_i + \epsilon_i = \begin{bmatrix} q \\ r \\ A_y \\ A_z \end{bmatrix}_{t=t_i} + \epsilon_i \tag{4}$$

where the accelerometer measurements are assumed to occur at the center of mass of the vehicle. This restriction is easily removed if necessary.

The estimation problem then involves determining v_0, w_0, q_0, r_0, and the appropriate aerodynamic coefficients from observations of the vehicle motion as given by Eq. (4) and subject to Eqs. (1). For most problems of interest the number of observations is greater than the number of unknown parameters and an additional criterion must be selected to uniquely determine the initial conditions on the state and the aerodynamic coefficients. For example, the observation error ϵ could be minimized by applying a least-squares approach to the quadratic cost function

$$\sum_{i=1}^{n} (y_i - G_i)^T W (y_i - G_i)$$

This is described in Ref. 1.

In the present study, a least-squares algorithm is used to compute the initial conditions and aerodynamic coefficients of a spinning ballistic vehicle from actual flight test gyro and accelerometer data. The square root variable metric (SRVM) optimization program described in Ref. 8 is used to compute the parameters.

Comparisons of the actual measurements, y_i, and the mathematical model of the data, G_i, are shown in Figs. 1 and 2. In the least-squares weighing matrix W, errors in accelerometer measurements were more heavily weighed than errors in the gyro data. This caused the predicted accelerometer data to match the actual data very well, as is shown in Fig. 1. In Fig. 2, however, the predicted gyro histories are substantially out of phase with the actual gyro data. Note that by changing the least squares weighing it is possible to be in phase with the gyro data and out of phase with the accelerometer data. Thus, the solution only shows that a phase difference exists in some components of the data but does not uniquely determine the instrument out of phase with the model. This phase difference not only affects the estimate for the initial conditions for Eqs. (1) but also causes estimates for the aerodynamic coefficients to be affected, as will be shown later. The scheme attempts to alter the coefficients to compensate for the phase differences. Thus, the standard approach, when applied to actual flight test data, often cannot correctly model the motion observed in the data. Some scheme to account for instrumentation induced phase shifts often must be used to correctly model the output of the instrument. This is described in the next section.

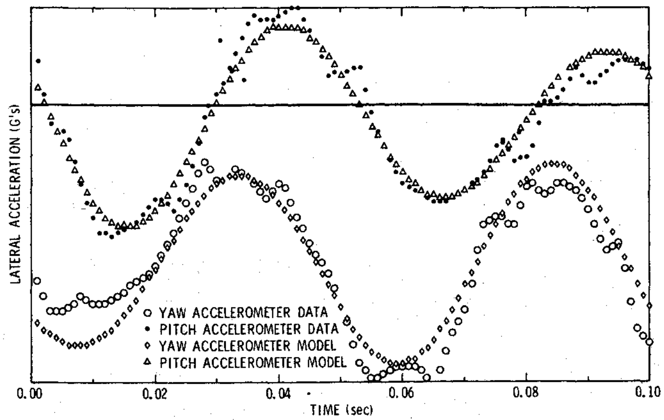

Fig. 1 Comparison of lateral accelerometer data and model for standard estimation scheme.

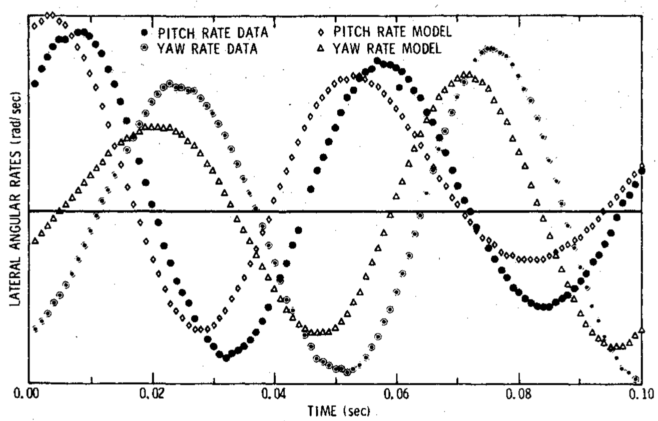

Fig. 2 Comparison of rate gyro data and model for standard estimation scheme.

Instrumentation Model

In Ref. 4, typical output plots of phase shift and amplitude modulation for a standard gyro receiving input rates over a large frequency range indicate that the behavior of the instrument can be well modeled as a second-order, linear, constant-coefficient differential equation. This should be expected since the dynamical models for gyros and accelerometers are basically second-order systems. When the forcing frequency of the instrument is a reasonable percentage of the natural frequency, it should be expected that phase shifts and amplitude modulation will occur. When different types of instruments are used, the natural frequencies will in general be different and relative phase shifts between the instruments will occur. In the case of a rolling vehicle, the lateral modes are coupled and there will be two frequencies in the motion of the vehicle. The instrument will also shift these frequencies relative to each other. Hence, it is not possible in this case to shift the gyro data relative to the accelerometer data and account for the phase shifts. The relative shift of the two frequencies forcing each instrument must be accounted for by each individual instrument. Thus, each instrument should be modeled to correctly account for all instrumentation induced data changes. This is easily accomplished by defining a z vector which satisfies the following differential equation:

$$\ddot{z} + \Lambda \dot{z} + \Omega z = \Omega G \tag{5}$$

The solution to this equation then models the output of an instrument which behaves like a second-order differential equation.

The matrices Λ and Ω are defined as

$$\lambda = \begin{bmatrix} 2\lambda_{RG} & 0 & 0 & 0 \\ 0 & 2\lambda_{RG} & 0 & 0 \\ 0 & 0 & 2\lambda_{LA} & 0 \\ 0 & 0 & 0 & 2\lambda_{LA} \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} \omega_{RG}^2 & 0 & 0 & 0 \\ 0 & \omega_{RG}^2 & 0 & 0 \\ 0 & 0 & \omega_{LA}^2 & 0 \\ 0 & 0 & 0 & \omega_{LA}^2 \end{bmatrix} \tag{7}$$

Here it is assumed that the natural frequency and damping of the two lateral motion gyros are the same and the natural frequency and damping of the two lateral accelerometers are the same. This is not necessary, of course, but is a good assumption for the example considered in this paper. The extension to different values of frequency and damping is trivial.

In order to incorporate Eq. (5) into the estimation scheme described earlier, the z vector is recognized as additional state variables and Eq. (5), written in first-order form, is added to Eqs. (1) to define the new dynamical system model. The state observation equation then becomes

$$y_i = z_i + \epsilon_i \tag{8}$$

At this point it would be possible to write Eq. (5) in first-order form, treat the initial conditions for z and $\dot{z}$ as unknown initial conditions, treat the values of λ and ω as unknown parameters, and solve the estimation problem just as before. This introduces four new parameters (λ_{RG}, λ_{LA}, ω_{RG}, and ω_{LA}) and eight initial conditions (z and $\dot{z}$) which must be estimated. Thus, the size of the vector to be estimated has substantially increased but the basic estimation problem is not altered. Also, estimating all of these additional parameters would probably allow the model to match the data but could degrade the estimates of the aerodynamic coefficients. It might be possible to match the data with almost any values for the coefficients. An approach which avoids this uniqueness problem is described below.

It is possible, with a few assumptions, to reduce the number of additional variables which must be estimated. If the natural frequency (or damping) of the instrument is known, it could be held at that constant value. If it is known that the instrument is, for example, 0.7 critically damped, then $\lambda = 0.7\omega$ and only the frequency could be estimated.

The large increase in the number of parameters to be estimated, however, comes from the initial conditions, so that it is not necessary to estimate them, would be very useful. This can be done if the linearized solutions of Eqs. (1), such as the solution shown in Ref. 6, accurately describes the motion of the vehicle over a very short time interval. If this is the case, then it is possible to write

$$G = C_1 \sin\omega_1 t + C_2 \cos\omega_1 t + C_3 \sin\omega_2 t + C_4 \cos\omega_2 t \tag{9}$$

where it is assumed that the ω_1 and ω_2 come from Ref. 6 and that the damping over a very short time interval is negligible. The vectors C_1, C_2, C_3, and C_4 are easily determined for given initial conditions for Eqs. (1). Thus, given an initial guess for v_0, w_0, q_0, and r_0, the linearized constant coefficient solution of Eqs. (1) is obtained. Knowing v and w or α and β from this solution it is possible to compute A_y and A_z. The C_1, C_2, C_3, and C_4 vectors can then be computed from the given initial conditions and the requirement that the solution shown in Eq. (9) be consistent with Eqs. (1). This involves inverting a 16×16 matrix to solve for the C_1, C_2, C_3, and C_4 vectors. This is easily done numerically. Once these are known then G is a known function of time. The steady-state solution of Eq. (5) is then easily obtained. If it is assumed that the instrument is sufficiently damped that the transient solution of Eq. (5) can be neglected, the steady-state solution becomes the total solution. Thus, the initial conditions corresponding

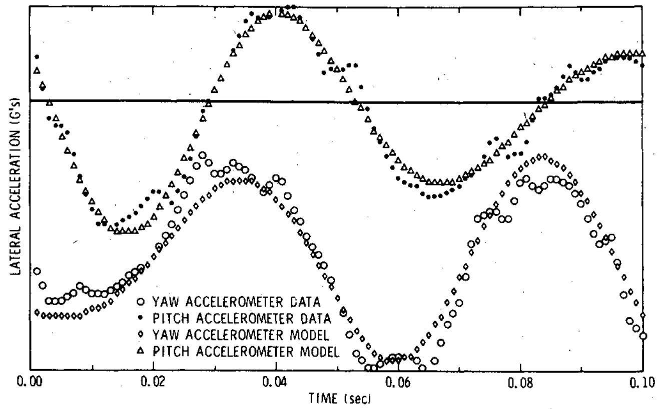

Fig. 3 Comparison of lateral accelerometer data and model using instrumentation modeling scheme.

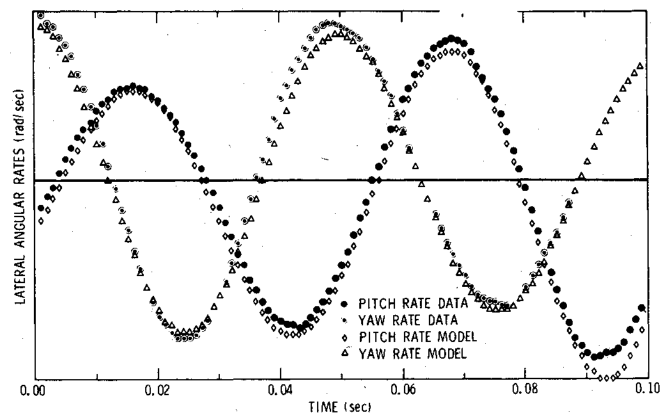

Fig. 4 Comparison of rate gyro data and model using instrumentation modeling scheme.

to the steady-state solution for z and $\dot{z}$ can be determined as a function of the initial conditions for the original state equations. This eliminates the necessity of estimating these eight parameters.

The estimation algorithm then involves guessing v_0, w_0, q_0, r_0, any aerodynamic coefficients to be estimated, and any instrumentation damping or frequency terms to be estimated. These are then corrected to cause

$$\sum_{i=1}^{n} (y_i - z_i)^T W (y_i - z_i)$$

to be a minimum. If the initial conditions for z and $\dot{z}$ are computed by the scheme described earlier, then during each iteration toward the solution a 16×16 matrix is numerically inverted to relate z_0 and $\dot{z}_0$ to the guesses for the original state variables. Relative to the total computing time, however, this inversion is insignificant.

Before the scheme described above was used to process the real data shown in Figs. 1 and 2, several sets of simulated data were processed by the estimation scheme to verify the method. The results from one of these simulations is shown in Table 1. The data were generated over a 0.1 s time interval using an instrument natural frequency of 200 rad/s for the lateral motion gyros and 400 rad/s for the lateral accelerometers. These results indicate that whenever phase shifts occur in measured data and they are not accounted for in an identification scheme, substantial errors can arise in the estimation of the aerodynamic coefficients. In addition, the simulated observations repeated the phase shift patterns seen in the real data of Figs. 1 and 2.

This instrumentation modeling scheme was then used to compute the motion from the flight test data described earlier.

Table 1 Comparison of estimated coefficients with and without the instrumentation modeling technique

Coefficient	True value	Estimated value, standard scheme	Estimated value, instrument modeling
C_{m_0}	0.01154	0.00948(−17.9)[a]	0.01175(1.8)
C_{n_0}	−0.01154	−0.00942(−18.4)	−0.01176(1.9)
C_{m_α}	−1.1032	−1.09625(−0.6)	−1.1056(0.2)
C_{z_α}	−1.4468	−1.7497(20.9)	−1.4242(−1.6)

[a] Number in parentheses represent the percentage difference between the estimate and the true value.

The scheme described to compute the initial values of z and $\dot{z}$ was used. The problem was initially solved assuming $\lambda = 0.7\omega_{RG}$, which the gyro handbook for this instrument indicates is correct. It was also solved by estimating both λ_{RG} and ω_{RG}. For this example the natural frequency and damping of the lateral accelerometer were well known and hence were not estimated. The frequency was well above that associated with the motion of the vehicle and hence the accelerometer data were not significantly affected by the instrument.

In summary, the problem was first solved by estimating ω_{RG} and the same parameters that were estimated when the results shown in Figs. 1 and 2 were obtained. For the second case λ_{RG} was also estimated. The results of the two cases were essentially identical, verifying that the instrument is essentially 0.7 critically damped. This along with the fact that the computed frequency agrees with the frequency quoted in the gyro handbook to within 15% implies that Eq. (5) is a good model of the instrumentation induced changes in the data. It also implies that the majority of the errors in the fit in Fig. 2 are caused by the instrumentation. The results of the fit are shown in Figs. 3 and 4. These figures indicate that the data are being modeled much better when Eq. (5) is included than they were without Eq. (5). Thus, the proposed instrumentation scheme does accurately model the phase shifts encountered in the data for this example. The aerodynamic coefficients computed using this scheme are all near the expected values. This was not true when the instrumentation modeling scheme was not incorporated into the estimation scheme.

Conclusions

Rate gyro and lateral accelerometer data from the flight test of a rolling vehicle were used to compute the motion of the vehicle. The standard estimation algorithm indicate that the data were not being accurately modeled and in particular large phase shifts existed between the gyro data and observation model. A simple second-order, linear, constant-coefficient differential equation model of the instrument was developed. This model was incorporated into the estimation scheme and the ability of the algorithm to accurately model the data was substantially improved. The scheme was used to estimate the aerodynamic coefficients from several simulated data sets. In general the results indicate that a failure to account for the phase shifts can result in estimates of the aerodynamic coefficients which may be at least 18% in error from the true values. When the instrumentation modeling scheme was used with actual flight test data, the estimates of the aerodynamic coefficients that were computed were near preflight predicted values. This was not true when estimates of the values were obtained without the instrumentation modeling scheme.

Acknowledgments

This work was supported by the U.S. Department of Energy.

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